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A Dynamically Efficient Crude Oil & Natural Gas Field Exploration and Development Contracting Equilibrium: Theory & Evidence

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PREFACE

The energy sector has been a dominant factor in Alberta's development and growth over the last half-century. The large capital investments and operating expenditures associated with finding and producing oil and gas have directly provided a major stimulus to the economy. But the indirect and induced impacts have been equally important. The development of many other industries supplying inputs to the energy sector, the generation of substantial export and government revenues, and the stimulus for large inflows of people have resulted in large 'multiplier' effects. In combination, these have also played a major role in shaping Alberta's 'character' which is generally distinguished by its highly educated, adjustable and entrepreneurial labour force, low unemployment and high labour force participation rates, strong work ethic and sense of self reliance, and its optimistic outlook.

In recent years the energy sector has become even more dominant and has increasingly made Alberta a key driver of the national economy. In a world with a rapidly growing demand for energy, having one of the largest concentrations of energy resources in the world might seem to translate into an assured, prosperous future. There is clearly huge potential associated with unconventional oil and gas, coal, remaining conventional resources and with alternative and renewable energy. However, translating this potential into reality will be daunting. Increasing constraints related to resource access, environmental impacts, infrastructure requirements, and availability of highly qualified people need to be addressed. Other challenges include the massive long-term investments in developing and implementing new technologies and making the right changes in the policy and regulatory framework. Indeed, the fact that relatively few nations have managed to convert resource wealth into high standards of societal welfare is a useful reminder of the magnitude of the challenges.

Alberta is in many respects at a crossroads. On the one hand complacency will almost certainly mean a dimming of the province's long-term prosperity. Declines in the conventional oil and gas sector will significantly dampen growth and prosperity. There are no other sectors of the province's economic base that could realistically expand sufficiently to offset significant declines in the dominant energy sector. On the other hand, visionary, strategic investments today can unlock non-conventional and other energy resources critical to securing a strong and prosperous long-term, sustainable future for the province.

It is in this context that ISEEE has undertaken a series of papers focused on Alberta's energy futures. The intent is to take a longer term look at the challenges, opportunities and choices and what they mean for Alberta's future. This first paper provides both a retrospective and a prospective overview of the impacts of the oil and gas sector. It is intended to frame and highlight the longer term issues and provide an anchor for more detailed analysis in subsequent papers.

A DYNAMICALLY EFFICIENT CRUDE OIL & NATURAL GAS FIELD EXPLORATION AND DEVELOPMENT CONTRACTING EQUILIBRIUM: THEORY & EVIDENCE[#]

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Abstract: *In the last fifty years, the United States accounted for over sixty percent of world-wide wells drilled, but less than fifteen percent of oil production. Facts such as these have led a number of authors to argue that crude oil and natural gas production in the United States is characterized by common property dissipation of rents. This paper argues that this view would be correct if the location of all crude oil and natural gas fields was known and that they need only be developed. But the fixed cost of exploration is necessary for field development. In order for exploration firms to recoup these fixed costs they must contract with all potential mineral rights owners in advance of exploration. An implication of our theory is that the cost of drilling an exploratory well includes the mineral rents, but these costs are sunk at the time development wells are dug. If mineral rents obey Hotelling's rule, the ratio of exploratory wells to development wells should decline over time. We test this prediction using a panel of states over the period 1955-2002 and find support for the hypothesis that the exploration and development markets for crude oil and natural gas has been efficient.*

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I. INTRODUCTION

In 1955 the United States accounted for forty-four percent of world crude oil production and about eighteen percent of world crude oil reserves. By 2002 it accounted for less than ten percent of world crude oil production and only about two percent of world crude oil reserves. This reduction occurred in spite of the fact that over the period 1955 to 1970, the United States accounted for over eighty percent of the crude oil and natural gas wells drilled in the world and still accounts for over forty percent of wells drilled. (See Fig. 1.)

A crucial institutional difference between the United States and the rest of the world is that mineral rights are owned mainly by individuals in the United States, while elsewhere they are mainly owned by the state.[1] Since mineral rights ownership patterns tend to be smaller than crude oil and natural gas field sizes, it is possible for there to be multiple owners of the mineral rights over a particular

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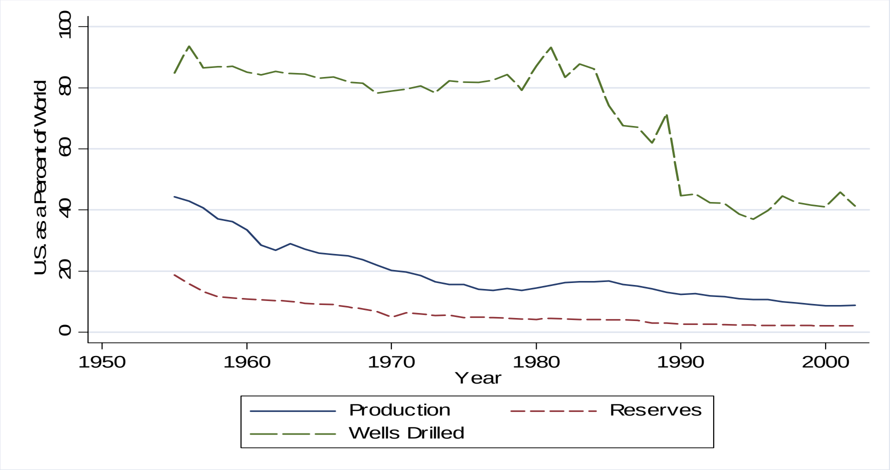
[1] Even in the United States, the federal government owns mineral rights on federal lands and states own mineral rights on riparian zones and three miles out into the ocean. There are also exceptions to state ownership in other countries (e.g., in Canada lands granted to the Hudson's Bay Company and to the railroads were fee simple, which meant they included mineral rights), but not to the extent of the United States.

field.[2] Libecap and Wiggins (1984, 1985a, 1985b), among others, have argued that contractual failures have occurred in the United States over how to allocate production on crude oil or natural gas fields. Thus it is possible that the decline in the U.S. share is because the U.S. squandered its resources through common property rent dissipation.

Fig. 1: United States Share of World Crude Oil Production, Crude Oil Reserves, and Crude Oil and Natural Gas Wells Completed, 1955-2002



Notes: Production and wells drilled (1970-2002) data is from the American Petroleum Institute’s *API Basic Petroleum Data Book* (Tables IV.1 and III.7, respectively). Wells drilled data prior to 1970 is from *World Oil* (February “Forecast and Review” issue). Reserves data is from the *Oil and Gas Journal*.

This paper challenges that view by deriving and testing a new model of contracting in crude oil and natural gas fields. The novel feature of the model is that we derive a simple competitive contracting equilibrium in a market where both exploration and development are essential to field production. Furthermore, exploration faces fixed costs that cannot be avoided, namely, seismic activities and the drilling of exploratory wells. We show that the only contracting equilibrium that survives in the presence of these fixed costs is one in which all mineral rights owners contract with the exploration firm prior to exploration. Since a single firm develops the field once discovered, rents are not dissipated. We then examine the dynamics of exploration and development in the context of this contracting equilibrium. We derive simple testable hypotheses about the dynamics of crude oil and natural gas field exploration and development which we find are supported by state level exploration and development drilling data over the period 1955-2002.

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[2] An example is the Exxon test area from the San Joaquin valley of California (Fig. 2.25 from Harbaugh et al. 1977). Many of the fields around the Bakersfield area are larger than the city of Bakersfield (population 250,000). Libecap and Wiggins (1985a) report that the West Texas Slaughterer oil field had 71,000 acres of surface area.

Under well defined property rights, a crude oil and natural gas exploration firm (a ‘wildcat’ operation) would begin its search for crude oil and natural gas by contracting with mineral rights owners over the distribution of rents *in advance* of drilling for crude oil or natural gas. The contract that maximizes the joint rents to the mineral rights owners and the wildcatter is one in which all potential mineral rights owners are included in the profit sharing agreement and the field is operated as a single unit (unitization). We assume that there is free entry into exploration and development and that owners of mineral rights are free to contract with either an exploration or a development firm. In equilibrium, it must be that no mineral rights owner wishes to switch to contracting with another development or exploration firm and that no exploration firm wishes to become a development firm or visa versa. Because exploration is a necessary precursor to development and because exploratory firms face fixed costs that other firms do not face, only a single exploratory firm contracts to do both the exploration and development on all properties over the field. This equilibrium satisfies both the fee entry condition for development and exploration firms, and the condition that no landowner can do better by voiding their current contract. Furthermore, it is efficient.

While we do not observe private contracts, we do observe drilling of both exploratory and development wells over time. Our empirical tests exploit the relationship implicit in the dynamic contracting equilibrium. Since the land yet to be explored is diminishing and a landowner may contract at most once, the acquisition cost to the wildcatter must rise over time in order for mineral rights owners to be indifferent between contracting their land now or at a point later in time (Hotelling 1931). Since the wildcatter must pay the land acquisition costs in advance of exploration, upon making a discovery, the wildcatter views the costs of land acquisition as sunk. Hence, when there is no contractual failure, the cost of drilling exploratory wells rises over time due to land scarcity, while drilling costs for development wells remain constant over time. Thus we expect the number of development wells to rise relative to the number of exploratory wells, all else constant.

Our empirical investigation uses a panel data of exploration and development in the twenty-five U.S. states for which crude oil and natural gas production, drilling, and reserves data are available in the post World War II era.[3] After controlling for changes in costs, prices, and probabilities of success, we find that the number of development wells per field is rising and the number of exploratory wells is falling, all else constant. This occurs in spite of the fact that the average size of fields has been diminishing over time. We conclude that the data support the basic properties of the efficient dynamic contracting equilibrium.

We follow Livernois and Uhler (1987) and Mendelsohn and Swierzbinski (1989) by assuming that production on a particular field depends upon the reserves remaining on that field. Development in our model occurs because

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[3] For the United States, data is available from several sources including *World Oil*, the *Oil & Gas Journal*, and the American Petroleum Institutes’ *Quarterly Well Completion Report*. By way of comparison, Canadian data from the Canadian Association of Petroleum Producers has total drilling, but does not break down the drilling data into exploratory and development drilling.

production also depends upon the number of wells drilled on the field. Since production declines as reserves are depleted, if an additional well is going to be placed on the field, it will be placed on the field as soon as the field is discovered. Thus each field's production decays at a rate that depends upon the number of development wells placed on that field at the time of discovery and upon how distant in time since the field was discovered. The current industry output is thus similar to the flow of services produced by a durable goods monopoly in that the equation describing aggregate production is an integral state equation.[4]

The model is stochastic in that both exploratory wells and development wells have a positive probability of being "dry", meaning that no discovery is made. However, given that the number of fields is quite large, it is not feasible to model the discovery process as stochastic dynamic programming model, since one would have to keep track of both the reserves and the number of wells placed on each field.[5] To overcome this problem, we assume a very simple form of a stochastic process which is due to Mendelsohn and Swierzbinski (1989). We assume a *proportion* of wells drilled in each instant are successful. This eliminates the jumps in the state variable, since the change in the number of fields or wells on a field is then proportional to the number of exploratory or development wells drilled, respectively.

In addition, since our focus is on a single country we also make a small open economy assumption about prices. Thus we solve for the exploration and development paths over time, but we do not solve for the equilibrium price path. In this sense, our model is more in the spirit of Gray (1914) than of Hotelling (1931).

It has commonly been believed that there are two stages the common property problem may present itself. At the development stage, Libecap and Wiggins (1984, 1985a, 1985b) and Yuan (2002) have found evidence that crude oil field development over-capitalization is at its lowest when informational asymmetries are at their lowest, which is before exploratory drilling begins. They found

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[4] Our model shares some similarities to the durable goods problem considered by Muller and Peles (1990) (see also Goering and Boyce, 1999). Unlike Muller and Peles, our model contains a Hotelling scarcity rent component due to the constraint on the total amount of land available to be explored. No such constraint is faced by a durable goods producer. See Kamien and Horner (2004) for more on the relationship between the durable goods problem and the exhaustible resource problem.

[5] Arrow and Chang (1982), *inter alia*, model exploration as a stochastic dynamic program. In their model, costs of extraction and production are both constant. Therefore, they do not have development of an crude oil field since there is no constraint on production. In addition, because exploration has a constant marginal cost, exploration occurs only when reserves are exhausted. The state equation for reserves "jumps" at each interval for which there is exploration and declines continuously in between these intervals. It is this jump that becomes infeasible to model when there are multiple state variables. Their model avoids the "curse of dimensionality" by collapsing the state variables to two: the area yet to be searched and the reserves on the field currently being exploited. In our model this is infeasible, because we need state variables for the number of fields, the reserves on each field, and the wells on each field. There were 53,975 named crude oil and natural gas fields in the United States in 2004 (see the "Crude oil and Natural gas Field Master List 2005", U.S. Department of Energy, Energy Information Administration, DOE/EIA-0370(05)).

evidence that states with a larger percentage of federal lands tended to have lowest rates of drilling per field, which they attributed to federal rules requiring unitization prior to exploration.[6] Our model shows that fixed costs of exploration create a *defacto* form of unitization, since only one firm develops the field.

In the exploration stage, the common property problem becomes a race for claming rights (e.g., Anderson and Hill 1982). We show that the race to claim rights does not dissipate rents because the mineral rights were allocated when the land was claimed. In virtually every case outside of Alaska, this occurred well before oil was discovered.[7]

Finally, there has been much attention in the literature on the informational spillovers from exploration (e.g., Hendricks and Porter 1989, 1996). As our focus is on both exploration and development, we assume the simplest information regime. Upon drilling an exploratory well, a wildcat firm and its rivals definitively know whether or not a deposit of oil or gas sits on an area of land. We assume this simple environment because, unlike Hendricks and Porter, we do not observe the location of successful wells beyond the state in which they are drilled in our aggregate data.

The remainder of the paper is organized as follows. Section II develops the basic theoretical model and derives comparative dynamics results to distinguish between the case where property rights are well defined and the case where property rights are poorly defined. Section III discusses the data from the United States over the period 1955-2002 and performs reduced form regression estimation on the panel data to test the hypotheses derived from the theoretical model. Section IV concludes.

II. THEORETICAL MODEL

In this section we develop a conceptual model of oil and gas field exploration and development. We begin by examining the economics of the development of a new field in subsection A. The main result is that the field is developed instantly upon discovery. In subsection B, we analyze the contracting problem on a field and show that with fixed costs to exploration firms the only contract that survives in a Nash equilibrium is one in which a single operator develops the field. Subsection C aggregates over fields and derives the necessary conditions that hold along the equilibrium paths for the number of exploratory and development wells per field. In subsection D, we characterize the equilibrium dynamics of the number of exploratory wells drilled, development wells drilled per field, and total wells drilled. Subsection E compares these equilibrium paths with the implied paths from previous literature. Subsection F characterizes the dynamics of aggregate production.

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[6] Kunce *et al*. (2002) argue that costs are higher on federal lands due to regulatory differences.

[7] Two acts of Congress were required before the 1968 Prudhoe Bay field could be brought to production. The Alaska Native Claims Act (1971) reallocated ownership of lands between federal, state, and Natives, and the Trans-Alaska Pipeline Act (1973) authorized the pipeline corridor.

A. Development of a New Field

We begin by describing the dynamics of production on a field.[8] Let $w(t)$ denote the number of wells placed on a field discovered at time t.[9] We assume that of the $w(t)$ wells drilled, a subset, $\phi \subset (0,1)$, are successful. Thus the number of producing wells on the field is $\phi w(t)$, which remains fixed over time. Let the quantity of recoverable reserves remaining on the field at time $s \geq t$ be denoted as $R(s)$. We assume that production on the field at time s is given by

$$q(s) = \phi w(t) R(s). \tag{1}$$

Equation (1) is the Schaeffer production function often utilized in natural resource models. The equation of motion for crude oil reserves on the field obeys

$$\frac{dR(t)}{dt} \equiv \dot{R}(t) = -q(t), \qquad R(0) = R_0 > 0. \tag{2}$$

Holding the number of wells fixed over time implies that the equation describing the stock at any instant in time $s \geq t$ is given by

$$R(s) = R_0 e^{-\phi w(t)(s-t)}, \quad \text{for } s \geq t, \tag{3}$$

From (1) and (3) we can express production at time $s \geq t$ as

$$q\big(s \mid w(t)\big) = \phi w(t) R_0 e^{-\phi w(t)(s-t)}, \qquad \text{for } s \geq t. \tag{4}$$

Therefore, the quantity produced depends only on the number of wells placed on the field at the time of discovery, the initial reserves on the field, and the distance in time since discovery, $s - t$. Field production is decreasing over time as reserves are depleted, since (4) implies that the rate of decay of production is $\dot{q}(s)/q(s) = -\phi w(t)$. Thus, field production exhibits exponential decay.

Let the expected future price at time $s > t$ be denoted as $p(s)$.[10] Then, the present value of rents to the crude oil field at the time of discovery is

$$PV_t = \int_t^{T_t} e^{-r(s-t)}[p(s)q\big(s \mid w(t)\big) - \phi C_m w(t)]ds - C_w w(t), \tag{5}$$

where T_t is the time at which a field discovered at time t is economically

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[8] See Kuller and Cummings (1974) and Black and LaFrance (1998) for models in which production depends upon inputs other than reserves that vary over time. We do not emphasize inputs other than the number of wells drilled as we cannot observe them in the aggregate data.

[9] We prove below (Lemma 2) that the field will be developed at the instant it is discovered.

[10] We assume that the expectation for the price does not change over time, i.e., $p(s|t) = p(s|t')$ for all t and $t' < s$. Hence we write the expectation only as $p(s)$.

exhausted given that $w(t)$ wells have been placed on the field, $p(s)$ is the expected price at time s, r is the discount rate, C_m is operating cost per successful well, and C_w is the drilling costs per well.

Assumption 1: The expected price at time $s > t$, given price $p(t)$ is $p(s) = p(t)e^{gt}$, where $g < r$.

Assumption 1 fixes the expectation on future prices to exhibit exponential growth. The condition $g < r$ is required for firms to wish to produce in the present. The shut-down condition for the field occurs when the term in the integrand in (5) vanishes. The shut-down point in time, T_t, satisfies $p(T_t)q\big(T_t \mid w(t)\big) - C_m w(t) = 0$. Under assumption 1, the number of periods the field operates satisfies

$$T_t - t = \frac{\ln\big(p(t)R_0 w(t)\big) - \ln(C_m)}{\phi w(t) - g}. \tag{6}$$

Note that as $C_m \to 0$ the quasi-rents in the first term of (5) are positive for all $s > t$. Furthermore, from (6), we see that as $C_m \to 0$, that $T_t \to \infty$.

Assumption 2: Maintenance costs are negligible, i.e., $C_m = 0$.

Assumption 2 implies that a successful well will be operated into perpetuity.[11] Taken together, Assumptions 1 and 2 imply that the stream of quasi-rents from the field is given by

$$V(w) = \int_t^{\infty} p(s)q\big(s \mid w(t)\big)e^{-r(s-t)}ds = \frac{p(t)R_0\phi w(t)}{r - g + \phi w(t)}. \tag{7}$$

Therefore, the value of the field depends upon the number of wells placed on the field at the time of discovery, the price at the time of discovery, the reserves in the field, and the rate of growth of the expected price path relative to the discount rate. The properties of the quasi-rents function are as follows:

Lemma 1: If the expected price at time $s \geq t$ is $p(s) = p(t)e^{g(s-t)}$ and maintenance costs are zero, then $V(w)$, the quasi-rent to the field, is an increasing concave function in the number of wells drilled on the field.

Proof: Simple calculations show the following:

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[11] In 2002, of the 319,176 wells in production in the United States, 84,698 wells were production rate class 1 wells, which averaged less than ¼ barrel of oil per day per well. (U.S. Department of Energy, Energy Information Agency, "Distribution and Production of Oil and Gas Wells by State," http://www.eia.doe.gov/pub/oil_gas/petrosystem/all-years-states.xls).

$$\frac{\partial V}{\partial w} = \frac{p(t)(r-g)R_0\phi}{\left(r-g+\phi w(t)\right)^2} > 0 \text{ and } \qquad \frac{\partial^2 V}{\partial w^2} = \frac{-2p(t)R_0\phi^2 w(t)}{\left(r-g+\phi w(t)\right)^3} < 0.$$

Q.E.D.

A key assumption in Lemma 1 is that the field is developed at the instant it is discovered. The next result shows that this is so:

Lemma 2: All of the development wells that are expected to be placed on the field are placed on the field at the moment of discovery.

Proof: See Appendix A.

The intuition behind Lemma 2 is that the cost of drilling wells is a constant to a particular field operator. Therefore, in the relevant range of the field, the investment rate is unbounded, which implies that all drilling occurs immediately.

We have assumed in the proof of Lemma 2 that the field is developed by a single operator. We now turn to demonstrating that this is so.

B. Oil and Gas Field Contracting Equilibrium

Here, we turn to the problem of deriving the equilibrium contract for crude oil and natural gas field exploration and development. We suppose that there are N mineral rights owners over each potential field, and that each mineral rights owner contracts with at most one producer. All contracts are written so that a fixed transfer from the producer to the mineral rights owner occurs at the beginning of the contract, and that there is no further sharing of profits.[12] There are two possible types of firms that may contract with mineral rights owners. Wildcat (exploration) firms both explore and develop the field, while development firms simply offer development services, contingent upon a discovery having occurred.

Let the probability of a discovery conditional upon exploration occurring be denoted as θ. Search in this model is very simple. Based on the results of a single exploratory well, the exploratory firm is able to determine whether oil or gas sits under the N mineral rights owner's land. We assume that once the result of this test well is known to the exploratory firm it becomes common knowledge. Because exploration is costly, we expect only one exploratory firm in equilibrium.

Let π_x and π_d denote the profits earned on each lease by a wildcat exploration firm and a development firm given that a discovery has been made; and let F_x and F_d denote the payment made to the mineral rights owner by the wildcat and development firms, respectively. Suppose that a wildcat exploration firm contracts with M mineral rights owners, where $1 \leq M \leq N$. Then his expected

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[12] These assumptions follow Yuan (2002), except that he allows mineral rights owners to contract with more than one developer. As will be seen, allowing mineral rights owners to contract with multiple development firms does not affect the equilibrium contracting outcome.

profits are

$$\Pi_x = M\big(\theta\pi_x - F_x\big) - C_w, \tag{8}$$

where the $-C_w$ term is the cost of drilling the exploratory well and the first term is the development profits.[13] Denote the mineral rights owner's profits (rents) as L_x and L_d, respectively, depending upon whether they contract with an exploration firm prior to exploration or with a development firm subsequent to a successful discovery. The M mineral rights owners who contract with the exploration firm each earn the fixed fee, F_x:

$$L_x = F_x. \tag{9}$$

In contrast, a mineral rights owner who chooses to wait and contract with a development firm earns expected profits (where the expectation is prior to exploration) of

$$L_d = \theta F_d. \tag{10}$$

The expected profits (again where the expectation is prior to exploration) to a development firm are thus

$$\Pi_d = \theta\big(\pi_d - F_d\big). \tag{11}$$

Following Yuan (2002), we assume that there is free entry into wildcat and development of crude oil fields. Thus, in equilibrium $\Pi_x = \Pi_d = 0$. Therefore, we conclude from (11) that $F_d = \pi_d$ and from (8) that $F_x = \theta\pi_x - C_w/M$. In order for mineral rights owners to be indifferent between writing a contract with a development firm conditional upon a discovery or writing a contract with an exploration firm prior to exploration it must be that $L_x = L_d$. This implies that the equilibrium number of mineral rights owners who contract with the exploration firm must satisfy

$$M = \frac{C_w}{\theta\big(\pi_x - \pi_d\big)}. \tag{12}$$

Note that (12) implies that π_x must be greater than π_d in order for M to be finite. Now, we are ready to determine π_x and π_d. Holding M fixed, the total number of wells drilled on the field is $w = Mw_x + (N - M)w_d$, where w_x and w_d are the Nash equilibrium choices of wells by the exploration firm and the $N - M$ symmetric development firms. Thus, π_x and π_d are given by

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[13] In (8), we specify C_w as the fixed cost of drilling an exploratory well and we assume that the exploratory well is not used for production. The same result as in Proposition 1 below can be obtained by assuming that the profits to the exploratory firm are written as $V_x = \theta M\pi_x - (1-\theta)C_w - MF_x$. In this specification, a successful exploratory well is productive, but with probability $1-\theta$, the result of exploration is a dry hole, with costs C_w.

$$\pi_x = \left(\frac{Mw_x}{w}\right)V(w) - C_w M w_x, \tag{13}$$

$$\pi_d = \left(\frac{w_d}{w}\right)V(w) - C_w w_d, \qquad d = M+1, \dots, N. \tag{14}$$

where $V(w)$ is given by (7). Recall there is only one exploration firm operating on each field. This firm has obtained the rights to develop M pieces of land. Mw_x in (13) therefore denotes the total number of development wells drilled by the exploration firm if successful at the exploration stage. As each development well produces the same flow of oil, the exploration firm's share of the present value of rents from the field, $V(w)$, is given by Mw_x/w. Each development firm in (14), on the other hand, operates on only one piece of land and therefore obtains a share w_d/w of $V(w)$. For both types of firms, the total cost of drilling is given by the drilling costs per well times the total number of development wells drilled.

The Nash equilibrium choices of numbers of development wells satisfy

$$Ms_x V'(w) + (1 - Ms_x)\frac{V(w)}{w} = C_w, \tag{15}$$

$$s_d V'(w) + (1 - s_d)\frac{V(w)}{w} = C_w, \qquad d = M+1, \dots, N. \tag{16}$$

where $s_x = w_x/w$ and $s_d = w_d/w$ are the share of total wells, w, on a mineral rights owner's property by an exploratory and by a development producer, respectively. Next, we impose symmetry among the development firms and let the shares sum to one:

$$Ms_x + (N - M)s_d = 1. \tag{17}$$

Equating (15) with (16) and solving the result and (17) simultaneously yields

$$s_d = \frac{1}{N - M + 1} \qquad \text{and} \qquad s_x = \frac{1}{M(N - M + 1)} = \frac{s_d}{M}. \tag{18}$$

This says that the exploration firm places a smaller number of wells on each property on which it purchases the mineral rights than do the development firms. This occurs because the exploration firm internalizes the externality it imposes on its other $M-1$ holdings. Since the development firms have no other holdings, they do not internalize any of the external costs they impose on other firms.

Given the values s_d and s_x from (18), we may deduce the equilibrium values of π_x and π_d from (13) and (14) to be

$$\pi_x = \pi_d = s_d\left[V(w) - C_w w\right]. \tag{19}$$

However, recall (12), which gives the equilibrium number of mineral rights owners with which the exploration firm must contract in equilibrium. Then (19) implies that $M \rightarrow \infty$, which is not possible given that M is constrained to be between 1 and N mineral rights owners. Thus, we conclude that no Nash equilibrium exists in which some mineral rights owners contract with an exploration firm while others wait to contract with a development firm if a discovery is made.

So what is the Nash equilibrium? It cannot be that all mineral rights owners wait to contract with development firms, as no development firms can exist if there does not exist an exploration firm. Thus the only possible Nash equilibrium is given by:

Proposition 1: All mineral rights owners contract with the exploratory firm prior to exploration.

In this case, $M = N$, $s_x = 1/N$ and $s_d = 0$. The equilibrium number of development wells is w^*, the number of wells that maximize net field rents. Because of free entry in the exploratory firms market, the equilibrium rents paid to each landowner are $F_x = \theta\pi_x^* - C_w/N$, where π_x^* is the equilibrium net profits given w^* wells.

Note that in contrast to claims in the literature, this result is efficient, in the sense that the exploration firm, if successful, chooses the number of wells to maximize the present value of the field. Why is the Nash equilibrium efficient? It is because of two related factors. First, the exploration firm is *necessary* in order to have production. Second, exploration firms face a fixed cost – the drilling of an exploratory well – which must somehow be recouped. What we have shown is that it is not possible for them to recoup their fixed cost when some mineral rights owners contract with other firms. Thus any equilibrium in which there is exploration and the fixed costs of exploration are recouped by the exploratory firm must have only one firm developing the field.

What could this Nash equilibrium fail? There are two obvious means. First, the landowners could attempt to hold up the exploratory firm. However, this is easily dealt with – all the exploratory firm has to do is write a contract that says each contract is valid only if *all* N landowners sign the contract. Second, it is possible that the contract could fail because the size of the field is larger than expected. In this case, even after contracting with N landowners, the exploratory firm could find itself in a situation where an additional $Z > 0$ landowners become potential competitors. However, even in this case, the exploratory firm is in a position to write the best deal with the additional Z landowners, since having two or more competitors lowers the total possible rents.

C. Dynamics of Exploration and Development

Next, we turn to the problem of determining how the aggregate number of exploratory wells, $x(t)$, and the number of development wells per field, $w(t)$, are jointly chosen over time.

Before continuing, note that the dissipation of rents at the exploratory stage is eliminated by the mineral rights owners. As most of the land in the U.S. was settled prior to the discovery of oil, we doubt that land owners dissipated the rents in an anticipation of oil discoveries.[14]

Recall that the proportion of exploratory wells that are successful is θ, and the proportion of development wells that are successful is ϕ. We assume for simplicity that θ and ϕ do not change over time. As fields are assumed to be identical, the number of development wells drilled on each new field discovered in period t is also identical, although $w(t)$ may vary over time. The number of exploratory wells drilled depends upon the cost of drilling an exploratory well, denoted as C_x, and the cost of mineral rights purchases, denoted as $F_x = \psi e^{rt}$, where ψ is the present value of mineral rights at time $t = 0$. The mineral rights purchase price must rise at the rate of interest in order for land owners to be indifferent between being paid for mineral rents now or at some later time; hence the current value of mineral rights rises over time according to Hotelling's rule.

In order for the mineral rental value, ψ, to be positive, the total amount of mineral rights available for leasing must be fixed. Let $A(t)$ denote the area of land which has not yet been explored by time t. With homogeneous land, $A(t)$ is given by[15]

$$A(t) = A(0) - \int_0^t Nx(s)ds, \tag{20}$$

where $A(0)$ is the total amount of land available to be searched, and N units are searched by each exploratory well drilled. The constraint on exploration is that $A(t) \geq 0$. It is this constraint that yields the positive mineral rights rental price, ψ. We assume that all land is equally likely to contain a mineral deposit, thus if any land is explored, all land will be explored.[16] Therefore, there exists some time $T > 0$ such that all land has been explored. At this point in time $A(T) = 0$, so that the total amount of land that has been searched is $A(0)$.

Total production at time t, which we denote as $Q(t)$, is the cumulative production from all past successful drilling. This can be written as

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[14] The first big discovery in Texas was the Spindletop well on January 10, 1901. Texas was settled prior to the Civil War. Discoveries in Oklahoma and California were also after they were settled. See note 7, *supra*, for a discussion of Alaska.

[15] Of course, (20) could be rewritten as $\dot{A}(t) = -Nx(t)$.

[16] It is well known (e.g., Mendelsohn and Swierzbinski 1989) that in a model of continuous grades of the resource that the scarcity rental value will rise at a rate less than the interest rate. For our purposes, what is important is that it rises over time. Nevertheless, we may conjecture that if lands were different in their expected grades of the stock, then the highest grade lands would be explored first, just as the highest quality stocks are exploited first.

(21) $$Q(t) = \int_0^t \theta x(s) q\big(t \mid w(s)\big) ds,$$

where, without loss of generality, we set $Q(0)$ to zero. This equation states that production at time t is the production from each of the $\theta x(s)$ fields discovered s periods in the past, of which $\phi w(s)$ wells are productive on each field. Notice that unlike (20), (21) cannot be differentiated to yield a closed form solution for $\dot{Q}(t)$ if $x(s)$ or $w(s)$ are changing over time. Thus, (21) requires that the optimization problem be stated as an integral state equation problem (e.g., Kamien and Muller 1976).

Let the total number of wells drilled at time t be denoted as

(22) $$D(t) \equiv x(t)\big[1 + \theta w(t)\big].$$

We assume that the costs of drilling depend on the total number of exploratory and development wells drilled and that the marginal cost of additional wells is increasing in the number of wells drilled. There are two empirical reasons to make this assumption. The first is that there is evidence that an increase in total drilling corresponds to an increase in average costs per well drilled (compare the spike in drilling in the early 1980s in Fig. 3 with the spike in average drilling costs in Fig. 6). The second is that if drilling costs were independent of the rate of drilling, then exploration would be "bang-bang." Fig. 3 however shows that there were tens of thousands of wells drilled in the United States every year between 1955 and 2002.

In the competitive equilibrium, exploration firms behave as though $C_x = \big(1 + \theta w\big)c'(D)$ and that $C_w = \theta x c'(D)$, since for every additional exploratory well $1 + \theta w$ development wells are dug and each additional development well is dug on each of the θx fields discovered.

Firms in the industry choose $x(t)$ and $w(t)$ to maximize

(23) $$\Pi = \int_0^\infty e^{-rt}\big[p(t)Q(t) - c\big(D(t)\big)\big]dt,$$

subject to the constraints (20) and (21). We let $\mu(t)$ and $\lambda(t)$ denote the multipliers for these constraints, respectively. Following Kamien and Muller (1976), the Lagrangian for this problem can be written as

(24) $$L = \int_0^\infty e^{-rt}\left[p(t)Q(t) - c\big(D(t)\big)\right]dt + \int_0^\infty \mu(t)\left(\int_0^t Nx(s)ds + A(0) - A(t)\right)dt$$

$$+ \int_0^\infty \lambda(t)\left(\int_0^t \theta x(s)q(t \mid w(s))\, ds - Q(t)\right) dt.$$

By changing the order of integration, the Lagrangian can be rewritten as

(25) $$L = \int_0^\infty \left\{H(t) + \mu(t)\left[A(0) - A(t)\right] - \lambda(t)Q(t)\right\}dt,$$

where the function $H(t)$ in (25) is given by

(26) $$H(t) = e^{-rt}\left[p(t)Q(t) - c\big(D(t)\big)\right] + Nx(t)\int_t^\infty \mu(s)ds$$

$$+ \theta x(t)\int_t^\infty \lambda(s)q\big(s \mid w(t)\big)\, ds.$$

The objective function in (25) can be solved using of calculus of variations in which x, w, Q and A appear in the integrand but in which $\dot{x}$, $\dot{w}$, $\dot{Q}$, and $\dot{A}$ do not appear (Kamien and Muller 1976). Thus, $Q(t)$ and $A(t)$ are chosen to satisfy

(27) $$\lambda(t) = e^{-rt}p(t)$$

and

(28) $$\mu(t) = 0.$$

From (27), the shadow value of additional output is the present value of the price. From (28), writing $\psi(t) \equiv \int_t^\infty \mu(s)ds$, then $\dot{\psi}(t) = -\mu(t) = 0$. This implies that the present value of the mineral rental price is a constant.

Using (27), we may write the first integral in (26) as $Nx\psi$ and using (28), we may write the second integral in (26) as $e^{-rt}\theta xV(w)$, where we drop the time notation on x and w except where necessary. Then the first-order-necessary conditions for $x(t)$ and $w(t)$ are given by

$$(29) \qquad e^{rt}H_x = \theta V(w) - N\psi e^{rt} - (1 + \theta w)c'(D) = 0$$

$$(30) \qquad e^{rt}H_w = \theta x\left[V'(w) - c'(D)\right] = 0.$$

The economic interpretation of (29) is that the present value of the marginal product of an additional exploratory well equals the sum of mineral rights purchase costs plus marginal drilling costs. The condition for development wells in (30) is similar, except that development wells do not require payment of mineral rights as these must be paid *prior* to exploration and that the present value of the marginal product of a development well includes the congestion costs on other wells as well as the direct marginal drilling costs.

The final condition that must hold in the dynamic equilibrium is that at time T, the point in time where $A(T) = 0$, the integrand in (25) vanishes:[17]

$$(31) \qquad c(D(T)) + Nx(T)\psi e^{rT} = \theta x(T)V(w(T)).$$

This condition says that the total costs of drilling in the final period equals the value of the fields discovered in that period. Note that if $x(T)$ is zero, then (31) is satisfied if, and only if, $c(0) = 0$. Conversely, if $x(T) > 0$, then combining (31) and (29), with the latter evaluated at time T, yields

$$(32) \qquad \frac{c(D(T))}{D(T)} = c'(D(T)).$$

This is the usual condition for the final period of operation of a mine (Gray 1914) when the average costs to the firm are U-shaped. This condition says that a price taking mine maximizes its rents in the last period by producing at the level where average costs are lowest. The only difference is that this condition is in terms of the inputs of wells drilled rather than in terms of output.

From (31) and (32) it is clear that whether or not $x(T)$ is equal to zero depends upon the nature of the cost function $c(D)$. When marginal cost is greater than average cost for all $D(t)$, then we get that $x(T) = 0$. When the average cost curves are U-shaped, we get that $x(T) > 0$. In both cases, $A(T) = 0$, since $\psi > 0$.

D. Equilibrium Dynamics of Exploration and Development

We now show how the number of exploratory and development wells changes over time. The system of equations (29) and (30) describe $x(t)$ and $w(t)$ in terms of time, the parameters, and the present value of the scarcity rental price, ψ, which is constant over time.

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[17] The calculus of variations condition for freely chosen T is that $F - z'F_{z'} = 0$ for all $z \in \{x,w,Q,A\}$, at T, where F is the integrand of (25). Since the $F_{z'}$ terms are all zero, this condition requires that $F = 0$ at T.

The main dynamic prediction of the model is given by Proposition 2:

Proposition 2: Holding all else constant, along the dynamic exploration and development equilibrium path, (*i*) the number of exploratory wells is decreasing over time, (*ii*) the number of development wells per field is increasing over time, and (*iii*) the total number of wells drilled is decreasing over time.

Proof: Time differentiating (29) and (30), holding ψ fixed, yields the rate at which the number of exploration wells drilled and the number of development wells per field change over time:

$$\begin{vmatrix} -(1+\theta w)c''(D) & -\theta D c''(D) \\ -\theta D c''(D) & \theta x\left[V''(w) - \theta x c''(D)\right] \end{vmatrix} \begin{vmatrix} dx \\ dw \end{vmatrix} = \begin{vmatrix} Nr\psi e^{rt} \\ 0 \end{vmatrix} dt. \tag{33}$$

Thus, by Cramer's Rule, we get that

$$\dot{x}(t) = \frac{[V''(w) - \theta x c''(D)]r\theta x N\psi e^{rt}}{-(\theta D)^2 c''(D)V''(w)} < 0, \tag{34}$$

$$\dot{w}(t) = \frac{r\theta D c''(D)N\psi e^{rt}}{-(\theta D)^2 c''(D)V''(w)} > 0. \tag{35}$$

The rate of change in total drilling, $D(t)$, is found using (22), (34) and (35):

$$\dot{D}(t) = (1+\theta w)\dot{x} + \theta x\dot{w} = \frac{(1+\theta w)V''(w)r\theta x N\psi e^{rt}}{-(\theta D)^2 c''(D)V''(w)} < 0. \tag{36}$$

Q.E.D.

In equilibrium, the number of exploratory wells is declining and the number of development wells per field is increasing, all else constant. This occurs because firms treat the mineral rights rents cost as sunk at the time the development decision is made. However, not only is the mineral rights cost not sunk at the time an exploratory well is dug, but the contracting equilibrium in part B above showed that before an exploratory well can be dug, mineral rents on all future expected development drilling must be paid. Thus expected exploratory drilling costs are $N\psi e^{rt} + (1+\theta w)c'(D)$, while expected development drilling costs are only $c'(D)$ (as the θx term does not affect the equilibrium in (29) when $w > 0$).

Finally, notice from (34)-(36) that convexity of the cost function $c(D)$ and concavity of the field quasi-rents function $V(w)$ are each required to ensure that the solution is not "bang-bang" in the aggregate.

E. Comparison with the Literature

The previous literature does not make clear what assumption is being made about exploratory drilling costs relative to development drilling costs. It seems clear that the equilibrium these authors have in mind is one where all producers' costs include mineral rents plus drilling costs.[18]

The arguments in Libecap and Wiggins (1984, 1985a, 1985b) do not explicitly include exploration – so there is no fixed cost as in the present paper – but the implication is that rents are driven to zero. If scarcity rents ψ are driven to zero, then the relative prices of $x(t)$ and $w(t)$ remain fixed over time. This implies that changes in parameters that affect $x(t)$ affect $w(t)$ (and therefore $D(t)$) similarly. Thus, the full rent-dissipation hypothesis is that $\dot{x}(t) = \dot{w}(t) = \dot{D}(t) = 0$.

Similarly, in Yuan (2002), while there are no fixed costs, rents are not driven to zero because the number of landowners is finite, so that $\psi > 0$. However, the implication is that producers pay ψ for each well – development or exploratory – that is drilled. In this case, the cost of development and exploratory wells each rise at the rate of interest, so that $\dot{x}(t) < 0$, $\dot{w}(t) < 0$, and $\dot{D}(t) < 0$ is implied.

F. Aggregate Production Dynamics

There has been much written in the popular press about "Hubbert curves," which are named after the geologist who predicted the year that production in the contiguous U.S. states would "peak." From (21), we may obtain an expression for the rate of change in aggregate output:

$$\dot{Q}(t) = \theta x(t) q\big(t \mid w(t)\big) + \int_0^t \theta x(s)\, \dot{q}\big(t \mid w(s)\big) ds. \tag{37}$$

The first term represents the additional production from new fields discovered at time t. The second term represents the rate at which production on all existing fields is declining. Note that at time $t = 0$, the first term dominates (37), as the integral is over zero fields, so that the rate of change in production is positive at $t = 0$. As $t \to \infty$, $x(t)$ vanishes, so only the second term, which is negative, remains. Thus (37) must necessarily yield a "Hubbert curve," where production peaks at some point in time $t \leq \infty$.

While most Hubbert curve ("peak oil") analysis is conducted by a simple curve fitting of past production, we have derived this result as an endogenous outcome of economic activity by optimizing agents.

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[18] Yuan (2002) explicitly makes this assumption.

III. UNITED STATES EXPLORATION AND DEVELOPMENT, 1955-2002

We now turn to an analysis of exploration and development in the United States over the post-War period. The data we analyze is a panel of state level data over the period 1955-2002. While there are thirty-three states with oil or gas production, reserves data is only available for twenty-five states.[19]

All drilling data prior to 1967 are from *World Oil*, and drilling data after 1967 are from *Quarterly Well Completion Report*, published by the American Petroleum Institute. Price, production, and reserves data are from the *API Basic Petroleum Data Book*, and cost of drilling data is from the *Joint Association Survey on Drilling Costs*, both published by the American Petroleum Institute. Price and cost data are in 1982 dollars, deflated using the Producer Price Index.

We begin by showing how the data varies across both time and geography in subsections A and B, respectively. Then subsection C reports the regression analysis methodology and results.

A. Geographical Variations in the Data

Table 1 presents the summary statistics by state for the data used in the model. As can be seen there is substantial variation across the states. Florida has averaged around fourteen wells drilled per year, while states like Kansas, Oklahoma and Louisiana have averaged over three thousand wells per year, and Texas has averaged over twelve-thousand wells per year.

The proportion of wells that are exploratory has ranged from two to three percent in the Appalachian states of Pennsylvania, West Virginia, and Ohio to over seventy percent in Florida.

Exploration success rates have also varied tremendously, ranging from lows of around eight percent successful in finding crude oil or natural gas in Florida and Illinois to fifty-five percent successful in West Virginia. Development success rates are higher than exploratory success rates in every state, and range from lows of less than sixty percent successful in Mississippi, and Nebraska to over ninety percent in Alaska, California, New York, Pennsylvania, and West Virginia.

Average drilling costs have also varied across states, with lows of less than one hundred thousand 1982 dollars in Kentucky and Illinois to a high of over two million per well in Alaska. Crude oil reserve additions per exploratory well drilled have ranged from 0.05 million barrels per exploratory well drilled in Indiana to over forty-four million barrels per exploratory well drilled in Alaska. Alaska also leads in the productivity per well, with crude oil production of over 346 barrels per well per year compared to ten states with less than one barrel per well per year in crude oil production.

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[19] The states that are dropped from the analysis are Arizona, Maryland, Missouri, Nevada, Oregon, South Dakota, Tennessee, and Virginia.

We control for the percent of federal landholdings by state in our regressions, since there is evidence that the federal government may control common property problems better than private mineral rights owners (e.g., Libecap and Wiggins 1985a, Kunce et al. 2002). The federal ownership of land may has ranged from less than one percent in New York to over eighty-six percent in Alaska.

Table 1: Descriptive Statistics by State, United States, 1955-2002

State	Total Wells Drilled, D_t	Exploratory Wells Drilled, x_t	Exploratory Percent of Wells Drilled	Exploratory Success Rate, θ (%)	Development Success Rate, ϕ (%)	Real Average Cost ($1000)	Federal Land Percent	Crude oil Reserve Additions per Exploratory Well	Crude oil Production per Well (bbl/yr)	Off-Shore Exploratory Percent
Alabama	142.4	41.5	29.1	14.4	74.2	437.3	3.41	0.37	19.55	0
Alaska	87.4	11.6	13.2	21.7	92.8	2,115.3	86.18	44.88	346.69	0
Arkansas	407.6	68.1	16.7	10.9	66.6	197.5	9.48	0.18	2.26	0
California	2,211.6	170.3	7.7	10.6	93.5	281.3	45.36	2.24	8.20	1.85
Colorado	969.6	304.2	31.4	16.7	72.1	225.7	35.97	0.10	9.11	0
Florida	14.0	10.2	73.1	8.1	76.5	716.4	10.15	2.23	81.37	0
Illinois	1,084.0	205.1	18.9	8.0	68.0	81.7	1.68	0.14	1.12	0
Indiana	355.6	98.9	27.8	10.5	60.5	54.5	1.97	0.05	0.96	0
Kansas	3,136.5	793.1	25.3	21.1	68.9	106.2	1.19	0.08	1.55	0
Kentucky	984.7	104.5	10.6	16.4	64.3	70.9	4.93	0.33	0.44	0
Louisiana	3,005.3	440.8	14.7	11.1	69.9	607.4	3.70	1.31	19.86	29.17
Michigan	558.5	152.2	27.2	12.6	67.6	200.6	9.97	0.11	3.89	0
Mississippi	408.0	156.1	38.2	10.8	57.4	413.1	5.27	0.26	15.13	0
Montana	472.8	169.5	35.9	15.7	73.9	211.2	29.25	0.18	7.19	0
Nebraska	226.8	126.9	55.9	11.6	57.6	120.1	2.04	0.06	5.32	0
New Mexico	1,315.0	162.9	12.4	33.4	88.6	319.1	33.74	0.56	5.25	0
New York	183.8	13.7	7.5	32.1	94.5	136.7	0.79	0.06	0.13	0
North Dakota	236.7	96.2	40.6	20.4	73.4	374.2	4.41	0.36	12.39	0
Ohio	1,434.9	46.0	3.2	30.6	85.1	136.7	1.20	0.17	0.43	0
Oklahoma	3,957.7	319.1	8.1	27.3	75.2	270.2	2.97	0.40	1.77	0
Pennsylvania	1,143.9	19.2	1.7	38.7	95.0	136.7	2.23	0.47	0.11	0
Texas	12,089.1	2,265.6	18.7	20.6	80.7	288.1	1.77	0.32	4.86	2.58
Utah	224.3	58.1	25.9	23.0	83.0	551.7	64.97	0.55	19.81	0
West Virginia	909.6	20.1	2.2	55.3	92.4	136.7	7.07	0.09	0.18	0
Wyoming	1,145.3	294.1	25.7	17.1	77.9	373.6	48.69	0.34	11.64	0
Un-weighted Sample Mean	1,404.7	263.5	29.8	22.6	75.8	299.8	19.1	2.00	24.53	1.24

Sources: Drilling statistics are from *World Oil* (1955-1966) and API's *Quarterly Well Completion Report* (1967-2002). Federal land percentages and population densities are from the *Statistical Abstract of the United States*. Crude oil reserve additions and production per well are estimated using data from API's *Basic Petroleum Databook.* All data are annual averages of the natural units. Blank means no data is available.

[a] Land density on private land is population per square mile on land not held by the federal government.

Similarly, we control for offshore percentage of exploratory wells to account for both differences in drilling costs and to capture the effects of strategic delays to bidding observed by Hendricks and Porter (1996). Louisiana, with almost thirty percent of drilling offshore, by far leads in this category.[20]

B. Temporal Variations in the Data

Fig. 3: Development and Exploratory Wells Drilled, and the Ratio of Development to Exploratory Wells Drilled, United States Total, 1955-2002

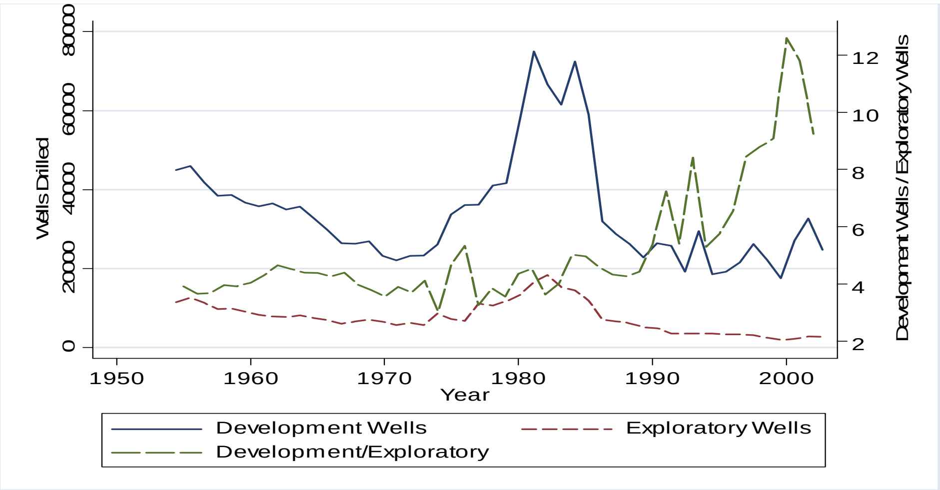
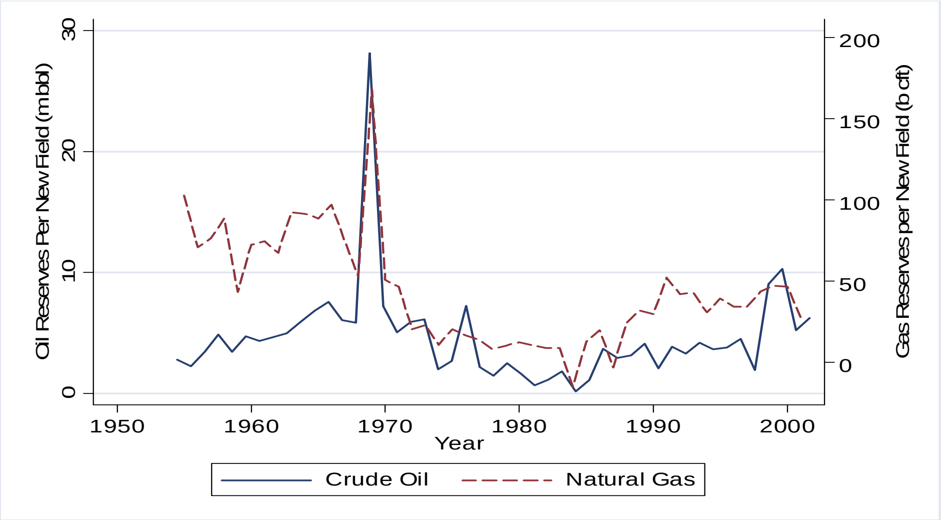


Fig. 4: Reserve Additions per New Field, United States Total, 1955-2002



Next, we turn to temporal variations in the data.[21] Fig. 3 shows the number of development and exploratory wells drilled in the United States and the ratio of development wells to exploratory wells for the 33 U.S. states with crude oil and

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[20] Data on offshore drilling by states like Alabama, Mississippi, and Florida were reported with less frequency, so these are ignored, although the percentages are quite small when reported.

[21] Data in Figs. 3 and 5-7 include the eight states that were dropped from the analysis (see *n*. 20, *supra*). Fig. 4 does not include this data for those eight states as it is unavailable.

natural gas production over the period 1955-2002. Both the number of development and exploratory wells experienced a dramatic jump in the period 1979-1984. The ratio of development wells to exploratory wells is rising on average over the entire sample, and is rising quite dramatically from the late 1980s onwards. Note that this is consistent with a first-best equilibrium in which low initial scarcity rents finally succumb to the pressures of exponential growth.

Fig. 4 shows the reserve additions per exploratory well of crude oil and natural gas. The spike in crude oil in 1968 is the Prudhoe Bay discovery. Note also that the minimum occurs near the end of the drilling spree in the early 1980s.

Fig. 5: Probability of Success for Development and Exploratory Wells, and Development/Exploratory Ratio of Probabilities, United States Total, 1955-2002

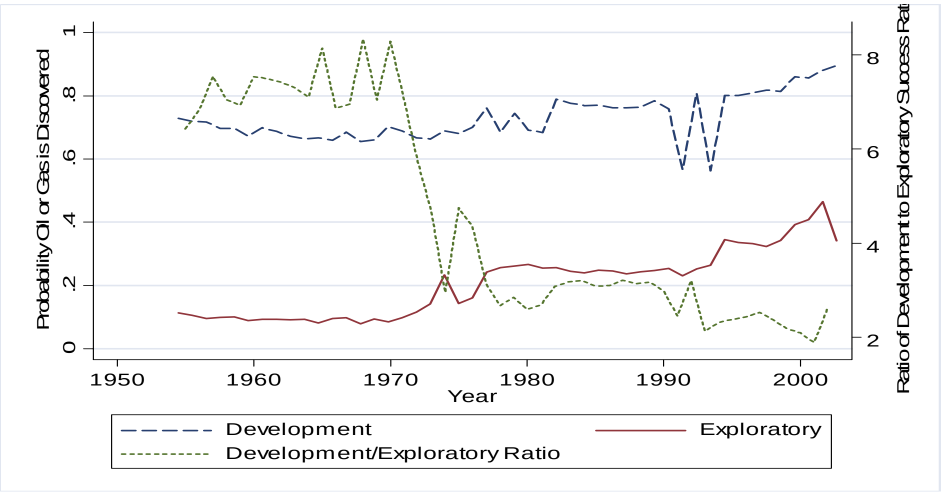


Fig. 6: Crude oil and Natural gas Prices, and Cost per Well Drilled, United States Annual Average, 1955-2002 (log scale)

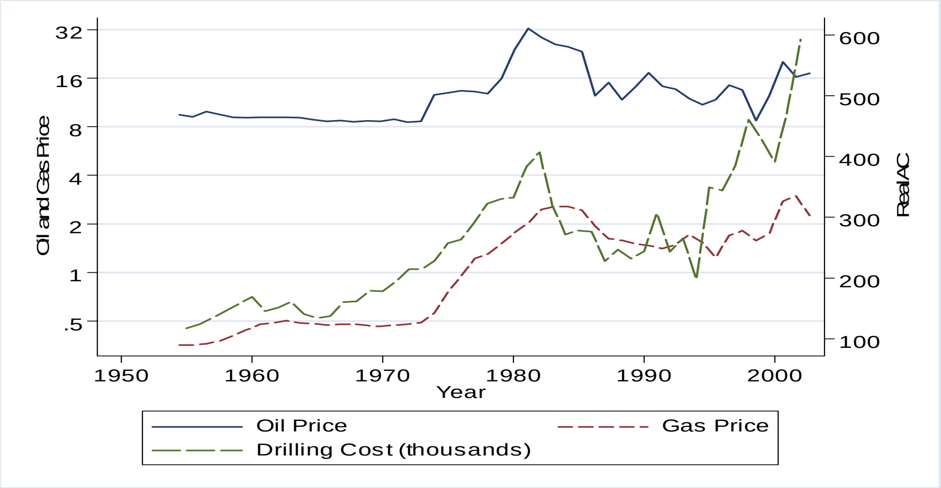
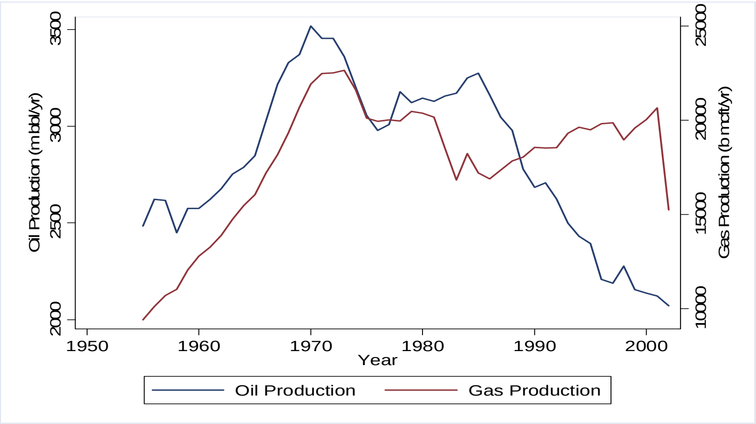


Figure 5 shows how the probability that a development or exploratory well is successful changed over time. Throughout the sample, exploratory wells are much less likely to be successful. However the most prominent feature of this graph is the jump in the probability of an exploratory discovery that occurred in the early 1970s. As with the changes in reserve additions, the shock to probabilities of success of exploratory wells appears to have been unanticipated, since it results in a regime change in the early 1970s.

Fig. 6 shows the paths of prices and costs of drilling (in 1982 dollars). The correlation in the price spikes in crude oil and natural gas is apparent, and it is also clear that the drilling boom in the early 1980s may have caused a spike in drilling costs. Drilling costs have also tripled since the early 1990s, perhaps due to an increase in offshore drilling.

Finally, in Fig. 7, we show the aggregate production for oil and gas over time. This indicates that the first big peak for oil occurred around 1970, with another spike in the mid-1980s. Natural gas follows a similar pattern.

Fig. 7: "Hubbert Curve" for U.S. Oil and Natural Gas Production



C. Regression Analysis

Now we turn to a regression analysis of the rates of exploration and development. It is possible that increase in the ratio of development to exploratory wells observed in Fig. 3 could be due to exogenous changes in the economic parameters, and so be consistent with the common property equilibrium. Thus, we shall inquire as to whether the pattern in Fig. 3 holds after controlling for changes in the economic parameters. To distinguish between these two possibilities, we estimate reduced form regressions of the numbers of exploratory, development and total wells drilled on the economic parameters.

Reduced form regression estimates can be obtained from the following equations:[22]

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[22] Variables other than the time trend that have only positive values are converted to log form so that the coefficients are elasticities. The coefficients β_{j6} are average growth rates over time. Reserve additions can be negative in years where reserves are adjusted downwards by a larger amount than the net additions.

$$\begin{aligned} (38) \qquad ln(x_{it}) = {} & \beta_{10} + \beta_{11} ln(\theta_{it}) + \beta_{12} ln(\phi_{it}) + \beta_{13} ln(p_t) \\ & + \beta_{14} ln(C_{it}) + \beta_{15} R_{it} + \beta_{16} t + \beta_{17} O_{it} + \beta_{18} F_{it} + \varepsilon_{1it} \end{aligned}$$

$$\begin{aligned} (39) \qquad ln(w_{it}) = {} & \beta_{20} + \beta_{21} ln(\theta_{it}) + \beta_{22} ln(\phi_{it}) + \beta_{23} ln(p_t) \\ & + \beta_{24} ln(C_{it}) + \beta_{25} R_{it} + \beta_{26} t + \beta_{27} O_{it} + \beta_{28} F_{it} + \varepsilon_{2it} \end{aligned}$$

$$\begin{aligned} (40) \qquad ln(D_{it}) = {} & \beta_{30} + \beta_{31} ln(\theta_{it}) + \beta_{32} ln(\phi_{it}) + \beta_{33} ln(p_t) \\ & + \beta_{34} ln(C_{it}) + \beta_{35} R_{it} + \beta_{36} t + \beta_{37} O_{it} + \beta_{38} F_{it} + \varepsilon_{3it}. \end{aligned}$$

Each of the variables in (38)-(40) correspond to definitions used in the theoretical model: x_{it} is exploratory wells, w_{it} is development wells per new field discovery, D_{it} is total wells drilled, θ_{it} is the probability of success at the exploratory level, ϕ_{it} is the probability of success at the development level, C_{it} is the average cost of drilling a well, R_{it} is the reserve additions per field, O_{it} is the offshore percentage, F_{it} is the percentage of federal land ownership, and t is time. The i subscripts denote the twenty-five states with crude oil or natural gas production, and t denotes the year, 1955-2002. We write each variable that varies across states and time with an '*it*' subscript, except prices which only vary across time. The ε_{jit}, $j = 1,2,3$, are assumed to be mean zero normally distributed random variables. We calculate w_{it} from the observed total development wells drilled, W_{it}, as $w_{it} = W_{it}/\theta_{it}x_{it}$.

The hypothesis to be tested has to do with the β_{j6} parameters. Proposition 2 predicts that $\beta_{16} < 0$, $\beta_{26} > 0$, and $\beta_{36} < 0$. Recall that in Libecap and Wiggins (1985a) model, the implied coefficients should all be zero and that in the Yuan (2002) model, $\beta_{16} < 0$, $\beta_{26} < 0$, and $\beta_{36} < 0$.

All regressions use a fixed effects panel regression method, where the errors are assumed to be of the form $\varepsilon_{jit} = \alpha_{ji} + e_{jit}$, where the α_{ij} corresponds to the fixed effect and e_{jit} corresponds to white noise. Since the data is time series data, we tested for unit roots. These results are contained in Table 3, reported in Appendix B. With the exception of the offshore percentage, all variables were stationary.

The econometric results are reported in Table 2. The main predictions are that the number of exploratory wells and the number of total wells are each decreasing over time, and the number of development wells per newly discovered field is increasing over time, all else constant. Each of these predictions is strongly supported by the data.

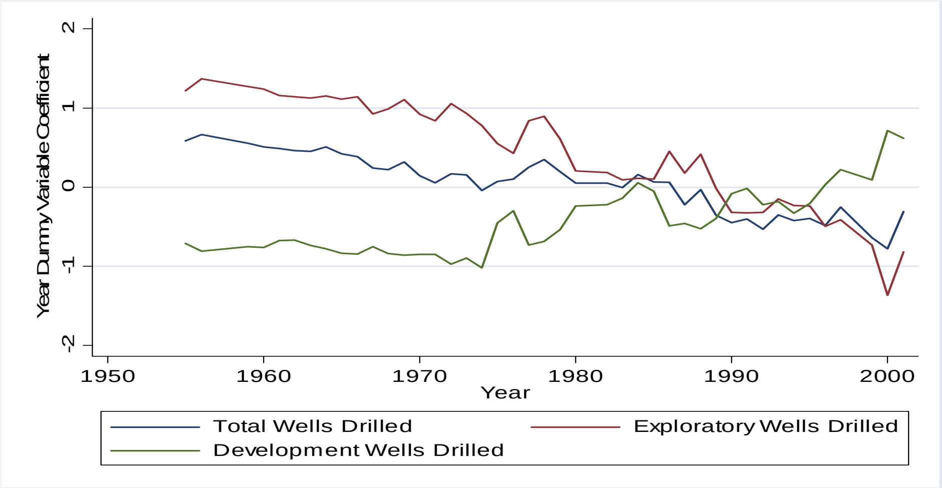
There are several other features of these regressions that are of interest. We see that drilling of exploratory wells is strongly influenced by the price, but the drilling of development wells is not affected by the price, although drilling costs do negatively affect the number of development wells drilled per field discovered.

Table 2: Fixed Effects GLS Panel Regression

Dependent Variable:	$\ln w(t)$	$\ln x(t)$	$\ln D(t)$
Year	0.017**	–0.040**	–0.018**
	(5.74)	(–18.73)	(–10.42)
Log of Real Oil Price (p_t)	–0.005	0.039**	0.038**
	(–1.29)	(13.57)	(16.0)
Log of Real Average Drilling Costs (C_{it})	–0.082*	0.006	–0.010
	(–2.20)	(0.27)	(–0.58)
Exploration Success Rate (θ_{it})	–3.860**	0.256	0.119
	(–19.57)	(1.86)	(1.36)
Development Success Rate (ϕ_{it})	–0.683**	0.370**	–0.129
	(–4.36)	(3.63)	(–1.46)
Reserve Findings per Field (R_{it})	0.001	0.000	0.000
	(1.51)	(–1.57)	(–0.43)
Offshore Percentage	–0.007	0.002	–0.003
	(–1.20)	(0.46)	(–0.95)
Federal Land Percentage	–0.008	–0.011	0.000
	(–0.49)	(–1.03)	(–0.05)
Number of Observations	1,012	1,016	1,016
Mean Observations per state	40.5	40.6	40.6
Log Likelihood Function Value	–719.7	–339.0	–117.2
Wald Test all $\beta = 0$ $\chi^2(32)$	980.8**	2550.3**	3614.3**
Autocorrelation estimated ρ	0.573	0.627	0.647
Wooldridge AR(1) test, F(1,24)	20.2**	22.7**	31.4**
Heteroskedasticity LRT, $\chi^2(24)$	752.7**	998.9**	1032.3**
State Dummies = 0, $\chi^2(24)$	609.9**	1765.7**	2863.3**

Notes: Asymptotic *t*-statistics in parentheses. "**" statistically different from zero at the 1% confidence level. "*" statistically different from zero at the 5% confidence level.

Fig. 8: Estimated Time Dummy Variable Coefficients



We were able to include the federal land percentage in the regressions since this varied over time as well as over state. However, as the federal land percentage varies much more by state than by time, this might bias our coefficient on federal land ownership downwards. Therefore, we plotted the state fixed effect coefficients against mean federal land percentage to see if there was a pattern. (The state fixed effects can be seen in Table 4 in Appendix C.) In none of the three cases did we discover a pattern and a linear regression of the fixed effect coefficients on the mean federal land percentage returned coefficients that were statistically zero. Thus, in contrast to both Libecap and Wiggins (1985a) and Kunce *et al.* (2002), we find no evidence in this aggregate data of influences due to federal land ownership.

Fig. 8 shows the estimated coefficients for the models (38)-(40) with time dummy variables rather than a time trend, with the remaining specification unchanged. The plots are of the estimated time dummy variable coefficients against time. Again, the pattern is that the time coefficient for exploratory and total wells drilled is decreasing and the time coefficient for development wells is increasing over the sample period.

V. DISCUSSION AND CONCLUSIONS

This paper derives and empirically tests an economic model of crude oil and natural gas field exploration and development. We argue that exploration, which is necessary for field development, is characterized by a fixed cost – the cost of exploration. In order for exploration firms to recoup these fixed costs, the competitive equilibrium requires that they must contract with all potential mineral rights owners in advance of exploration. Thus, we argue that crude oil and natural gas development in the United States has been efficient.

An implication of our theory is that the cost of drilling an exploratory well includes the mineral rents, but that these rents are sunk at the time development wells are dug. If mineral rents obey Hotelling's rule, the ratio of exploratory wells to development wells declines over time. We tested this prediction using a panel of states over the period 1955-2002 and find broad support for the hypothesis that the exploration and development markets for crude oil and natural gas has been efficient over this period.

The model we have considered makes three important assumptions. We discuss these here. First, the theoretical model ignores variation in the size of fields. In the empirics, we control for the average size of discoveries in each state and each period, which controls for unexpected changes in field size.[23] However, a more important implication of unexpected changes in field sizes is that it may upset the contracting equilibrium derived in part II.B. If the field is unexpectedly larger than the area contracted for, the exploration firm will face rivals in the development stage. This would cause the exploration firm to earn subnormal profits, which would cause it to go bankrupt. If this happened regularly enough in the data, it may cause a distortion in the allocation of development to exploratory wells. However, our empirics find no such distortion. Related to this point, note

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[23] Note that all comparative dynamics results must be interpreted as *unexpected* changes in the exogenous variable.

that most of the previous research on crude oil field development uses data from the pre-World War II era.[24] This era was characterized by unusually large fields.[25] Thus, it is more likely that fields larger than the expected field may have been discovered during this time period, especially given the state of geological knowledge. However, without an analysis on data from that era such as we have conducted, we can only speculate that this is the difference in results.[26]

We also assumed that neither the probability of discovery nor the size of fields changes as prior discoveries are made. The theoretical implication of having a single "grade" of field is that scarcity rent rises at the rate of interest. If quality of fields is diminishing over time, then the scarcity rental value would rise at *less* than the rate of interest (e.g., Mendelsohn and Swierzbinski 1989). However, the scarcity rental value would still be rising in any Hotelling model. Thus, this assumption should have only a quantitative, not a qualitative, effect upon the empirical results.

The final area that we ignore is technological change. Our primary empirical prediction that the ratio of development to exploratory wells is increasing over time could result from technological change that is biased in favor of exploration. However, many of the technological changes – such as "horizontal drilling," which increases the surface area of pipe exposed to the reservoir – are improvements to development drilling.

Note that while our empirical tests suggest that Hotelling scarcity effects are at work, they do not tell us how large is the scarcity rental value, only that it is large enough to affect the time paths of the numbers of development and exploratory wells. It is entirely possible, however, that rents have been dissipated, though not completely. Our analysis cannot rule out that possibility.

Finally, what do our results suggest is the answer to the decline in the relative status of the U.S. as a producer of oil and gas in the world? Our results suggest that it cannot be blamed on the property rights institution for mineral rights. A more likely answer is simply that the U.S. is much farther along its development path than is the rest of the world with respect to oil and gas. As of 2005, there were 53,975 oil and gas fields in the United States (with a median discovery date of 1965), while the rest of the world has discovered 4,423 fields (median discovery date of 1976).[27] As it is unlikely that the U.S. is better endowed with reserves than the rest of the world, it is likely that there remains much more oil to be found – although perhaps not much more in the United States.

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[24] Yuan (2002), for example, cites data from 1938-40. Libecap and Wiggins (1984) focus on fields in Oklahoma and Texas during the period 1926-35.

[25] The median year of discovery for a field in the United States is 1965 (see "Crude oil and Natural gas Field Master List (DOE/EIA-0370(05)), *opt cit.* at note 5. However, for the subset of fields for which the total recoverable reserves are estimated to be greater than one-hundred million barrels, the median date of discovery is 1938 (see "US Fields with Reserves Exceeding 100 Million Bbl., *Oil and Gas Journal*, January 25, 1988, at p. 60).

[26] Larger fields would result in larger numbers of wells per field, since $dV'(w)/dR_0 > 0$.

[27] The data on world fields is from the *Oil and Gas Journal*'s annual compilation of World Oil Production (December issue, various years).

APPENDIX A: PROOF OF LEMMA 2.

Without loss of generality, let the period of discovery be normalized to zero. We shall let $g = 0$, although the proof easily extends to $g \neq 0$. Given assumptions 1 and 2, the firm chooses the investment rate in development wells, $I(t) \geq 0$, to maximize

$$\text{(A.1)} \qquad \int_0^\infty e^{-rt}\left[pR(t)\phi w(t) - C_w I(t)\right]dt,$$

subject to the equations of motion

$$\text{(A.2)} \qquad \dot{w}(t) = I(t), \qquad w(0) = 0,$$

$$\text{(A.3)} \qquad \dot{R}(t) = -\phi w(t)R(t), \qquad R(0) = R_0.$$

Thus, we treat $w(t)$ as the stock of wells on the field. Let $\nu(t)$ and $\chi(t)$ denote the multipliers for (A.2) and (A.3), respectively. Then the Hamiltonian for this problem is

$$\text{(A.4)} \qquad \Omega(t) = e^{-rt}\left[pR(t)\phi w(t) - C_w I(t)\right] + \nu(t)I(t) - \chi(t)\phi w(t)R(t).$$

The first-order-necessary conditions include

$$\text{(A.5)} \qquad -e^{-rt}C_w + \nu(t) \leq 0 \qquad (= 0 \text{ when } I(t) > 0),$$

$$\text{(A.6)} \qquad \dot{\nu}(t) = -e^{-rt}p\phi R(t) - \chi(t)\phi R(t),$$

$$\text{(A.7)} \qquad \dot{\chi}(t) = -e^{-rt}p\phi w(t) - \chi(t)\phi w(t).$$

Notice that (A.5) does not contain $I(t)$, so the solution is "bang-bang." Furthermore, $I(t)$ is unbounded from above, so that when investment occurs, it does so at a maximal rate. From (A.3), we know that in any interval where $w(t)$ is fixed, that $R(t) = R_0 e^{-\phi wt}$. Solving (A.7) in this interval yields

$$\text{(A.8)} \qquad \chi(t) = \frac{e^{-rt}p\phi w}{r + \phi w} + k_1,$$

Where k_1 is an unknown constant. The transversality condition is that

$$\text{(A.9)} \qquad \lim_{t\to\infty} R(t)\chi(t) = 0,$$

From which it may be deduced that $k_1 = 0$. Using (A.8) in (A.6), we may solve for $\nu(t)$:

$$\text{(A.10)} \qquad \nu(t) = \frac{e^{-(r+\phi w)t}p\phi R_0 r}{(r + \phi w)^2} + k_2.$$

If investment is positive at time $t = 0$, then $\nu(0) = C_w$ from (A.5). Therefore, the full solution to $\nu(t)$ is

$$\nu(t) = C_w - \frac{p\phi R_0 r(1 - e^{-(r+\phi w)t})}{(r + \phi w)^2}. \tag{A.11}$$

All that remains is to show that if $\nu(t)$ satisfies (A.11), then $\dot{\nu}(t) < 0$. Simple calculation shows that this is so:

$$\dot{\nu}(t) = -\frac{p\phi R_0 r e^{-(r+\phi w)t}}{r + \phi w} < 0. \tag{A.12}$$

If $g \neq 0$, the solution is found by substituting $r - g$ for r in the above calculations. *Q.E.D.*

APPENDIX B: UNIT ROOT TESTS.

We conducted a Dickey-Fuller unit root test on the variables used in the regression by regressing

$$y_t = \gamma_0 + \gamma_1 y_{t-1} + \varepsilon_t \tag{B.1}$$

As the data was panel data, we omitted observations where the state in period t did not equal the state in period t–1. The unit root tests are given in Table 3.

Table 3: Unit Root Tests on Variables Used in Regressions

Variable	$\hat{\gamma}_1$	*s.e.*($\hat{\gamma}_1$)	Dickey-Fuller Statistic	Observations
Log of Exploratory Wells Drilled (x_{it})	0.958	0.009	-4.45**	1081
Log of Total Wells Drilled (D_{it})	0.976	0.007	-3.63**	1084
Log of Development Wells Drilled per Field (w_{it})	0.810	0.018	-10.62**	1011
Log of Real Oil Price (p_t)	0.856	0.016	-9.27**	1103
Reserve Findings per Field (R_{it})	0.287	0.029	-24.46**	1079
Log of Real Average Drilling Costs (C_{it})	0.895	0.014	-7.78**	1017
Log of Exploration Success Rate (θ_{it})	0.829	0.018	-9.40**	1011
Log of Development Success Rate (ϕ_{it})	0.609	0.024	-16.17**	1077
Federal Land Percentage (F_{it})	0.994	0.002	-3.56**	1103
Offshore Percentage (O_{it})	0.982	0.010	-1.78	1103
Offshore Percentage (when > 0)	0.954	0.031	-1.50	126

Notes: The 1% critical value is -3.42. $\hat{\gamma}_1$ is the estimated γ_1. *s.e.*($\hat{\gamma}_1$) is the standard error of the estimate. "**" statistically different at the 1% confidence level. "*" statistically different at the 5% confidence level.

As is clear from the data, the null hypothesis that $\hat{\gamma}_1 = 1$ is rejected for all but offshore percentage.

APPENDIX C: STATE FIXED EFFECTS.

Table 4 reports the state fixed effect coefficients for the models in Table 2.

Table 4: State Fixed Effect Coefficients and Significance

State	$\ln w(t)$	$\ln x(t)$	$\ln D(t)$
Constant (Alaska)	-26.903**	81.138**	40.643**
	(-4.56)	(18.99)	(11.32)
Alabama	-1.449	0.477	0.667
	(-1.03)	(0.49)	(0.8)
Arkansas	-0.664	0.824	1.417
	(-0.51)	(0.92)	(1.87)
California	0.856	2.072**	3.165**
	(1.1)	(3.9)	(7.03)
Colorado	-1.342	2.556**	2.323**
	(-1.49)	(4.08)	(4.39)
Florida	-1.811	-0.821	-1.174
	(-1.36)	(-0.89)	(-1.44)
Illinois	-0.668	1.831	2.379**
	(-0.47)	(1.85)	(2.87)
Indiana	-1.291	1.099	1.223
	(-0.9)	(1.11)	(1.47)
Kansas	-1.562	3.140**	3.420**
	(-1.09)	(3.19)	(4.07)
Kentucky	-0.465	1.264	2.263**
	(-0.34)	(1.3)	(2.75)
Louisiana	-0.079	2.525**	3.512**
	(-0.06)	(2.63)	(4.35)
Michigan	-1.305	1.592	1.755*
	(-1)	(1.78)	(2.32)
Mississippi	-1.789	1.574	1.406
	(-1.31)	(1.67)	(1.77)
Montana	-1.418	1.887**	1.570**
	(-1.41)	(2.69)	(2.67)
Nebraska	-2.720	1.352	0.764
	(-1.91)	(1.38)	(0.91)
New Mexico	-0.065	1.801**	2.597**
	(-0.07)	(2.78)	(4.77)
New York	1.043	-1.479	0.956
	(0.65)	(-1.22)	(1.08)
North Dakota	-1.809	1.000	0.849
	(-1.3)	(1.05)	(1.05)
Ohio	1.002	0.298	2.692**
	(0.69)	(0.29)	(3.22)
Oklahoma	-0.081	2.215*	3.676**
	(-0.06)	(2.29)	(4.51)
Pennsylvania	1.943	-0.674	2.478**
	(1.35)	(-0.66)	(2.99)
Texas	-0.937	4.142**	4.810**
	(-0.66)	(4.24)	(5.83)
Utah	-0.820	1.193**	0.829*
	(-1.48)	(3.07)	(2.46)
West Virginia	1.613	-0.405	2.196**
	(1.18)	(-0.42)	(2.81)
Wyoming	-0.897	2.674**	2.522**
	(-1.23)	(5.28)	(5.84)

Notes: Alaska is the omitted state. Asymptotic *t*-statistics in parentheses. "**" statistically different from zero at the 1% confidence level. "*" statistically different from zero at the 5% confidence level.

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